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B to Light Tensor Meson Form Factors Derived from Light-Cone Sum Rules

Kwei-Chou Yang*

Department of Physics, Chung Yuan Christian University, Chung-Li, Taiwan 320

Abstract

Using the recent results for the two-parton light-cone distribution amplitudes of the tensor meson, we calculate the form factors for the decays of $B_{u,d,s}$ into the light $J^{PC} = 2^{++}$ tensor mesons via the vector/axial-vector/tensor current with the light-cone sum rules. We also obtain the q^2-dependence of the form factors.

* kcyang@cycu.edu.tw

1. Semileptonic and radiative B decays, which involve light tensor mesons and are related to $b \to s(d)$ transitions, contain rich phenomena relevant to the standard model and new physics. It is interesting to note that due to the G-parity, both the chiral-even and chiral-odd two-parton light-cone distribution amplitudes (LCDAs) of the light $J^{PC} = 2^{++}$ tensor meson are antisymmetric under the interchange of the momentum fractions of its *quark* and *anti-quark* in the SU(3) limit [1]. Therefore, tensor mesons cannot be produced from the local $V \pm A$ currents of the standard model. The two-body B decays involving tensor mesons are of great interest because these decays can further shed light on the underlying helicity structure [2].

In this paper we calculate the form factors for the $B_{u,d,s}$ decays into light $J^{PC} = 2^{++}$ tensor mesons (T) via the vector/axial-vector/tensor current in the light-cone sum rule (LCSR) approach. These form factors are relevant to exclusive B decays involving the tensor meson in the final state. Some model calculations for these form factors can be found in the literature [3–7]. In the quark model language, the $J^{PC} = 2^{++}$ tensor meson is described by a constituent quark-antiquark pair with angular momentum $L = 1$ and total spin $S = 1$. The observed tensor mesons $f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ and $K_2^*(1430)$ form an SU(3) $1\,^3P_2$ nonet. Although light-cone sum rules have been widely used to calculate various form factors, $B \to$ tensor meson form factors have not been systematically studied [1]. The relevant inputs in this study are the LCDAs of the tensor mesons. However, only chiral-even two-parton LCDAs for the $f_2'(1525)$ were given about ten years ago [9]. Recently, we have systematically studied the chiral-even and chiral-odd two-parton LCDAs for $J^{PC} = 2^{++}$ tensor mesons [1].

2. For a tensor meson with mass m_T and four-momentum $(E, 0, 0, P_3)$ moving along the z-axis, its polarizations $\epsilon^{\mu\nu}_{(\lambda)}$ with helicity λ can be represented in terms of the polarization vectors [10]

$$\varepsilon(0)^{*\mu} = (P_3, 0, 0, E)/m_T, \quad \varepsilon(\pm1)^{*\mu} = (0, \mp1, +i, 0)/\sqrt{2}, \tag{1}$$

and are given by

$$\epsilon^{\mu\nu}_{(0)} \equiv \sqrt{\frac{1}{6}}[\varepsilon(+1)^\mu\varepsilon(-1)^\nu + \varepsilon(-1)^\mu\varepsilon(+1)^\nu] + \sqrt{\frac{2}{3}}\varepsilon(0)^\mu\varepsilon(0)^\nu, \tag{2}$$

$$\epsilon^{\mu\nu}_{(\pm1)} \equiv \sqrt{\frac{1}{2}}[\varepsilon(\pm1)^\mu\varepsilon(0)^\nu + \varepsilon(0)^\mu\varepsilon(\pm1)^\nu], \quad \epsilon^{\mu\nu}_{(\pm2)} \equiv \varepsilon(\pm1)^\mu\varepsilon(\pm1)^\nu. \tag{3}$$

The polarization tensors $\epsilon^{(\lambda)}_{\alpha\beta}$ are symmetric and traceless. Moreover, they satisfy the divergence-free condition $\epsilon^{(\lambda)}_{\alpha\beta}P^\beta = 0$ and the orthonormal condition $\epsilon^{(\lambda)}_{\mu\nu}(\epsilon^{(\lambda')\mu\nu})^* = \delta_{\lambda\lambda'}$.

By considering the semileptonic B decays involving tensor mesons, we can define the form factors: [5–7]

$$\langle T(P,\lambda)|\bar{q}\gamma_\mu b|B(p_B)\rangle = -i\frac{2}{m_B + m_T}\varepsilon_{\mu\nu\alpha\beta}e^{*\nu}_{(\lambda)}p_B^\alpha P^\beta V^{BT}(q^2), \tag{4}$$

$$\langle T(P,\lambda)|\bar{q}\gamma_\mu\gamma_5 b|B(p_B)\rangle = (m_B + m_T)e^{(\lambda)*}_\mu A_1^{BT}(q^2) - (e^{(\lambda)*}\cdot p_B)(p_B + P)_\mu\frac{A_2^{BT}(q^2)}{m_B + m_T}$$

[1] In [8], the $T_1(0)$ form factor for the $B \to K_2^*$ transition was calculated by using the LCSR approach. However, the assumed leading-twist LCDA for the tensor meson does not match its G-parity property.

$$-2m_T\frac{\epsilon^{(\lambda)*}\cdot p_B}{q^2}q^\mu\left[A_3^{BT}(q^2)-A_0^{BT}(q^2)\right], \tag{5}$$

$$\langle T(P,\lambda)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = 2T_1^{BT}(q^2)\varepsilon^{\mu\nu\rho\sigma}p_{B\nu}P_{T\rho}e^*_\sigma, \tag{6}$$

$$\begin{aligned}\langle T(P,\lambda)|\bar{q}\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle &= -iT_2^{BT}(q^2)\left[(m_B^2-m_T^2)e^{*\mu}-(e^*\cdot p_B)(p_B^\mu+P_T^\mu)\right]\\ &\quad -iT_3^{BT}(q^2)(e^*\cdot p_B)\left[q^\mu-\frac{q^2}{m_B^2-m_T^2}(p_B^\mu+P_T^\mu)\right],\end{aligned} \tag{7}$$

where $q_\mu=(p_B-P)_\mu$ and $e^{*\mu}_{(\lambda)}\equiv\epsilon^{*\mu\nu}_{(\lambda)}q_\nu/m_B$. We adopt the convention $\epsilon^{0123}=-1$, and

$$A_3^{BT}(q^2)=\frac{m_B+m_T}{2m_T}A_1^{BT}(q^2)-\frac{m_B-m_T}{2m_T}A_2^{BT}(q^2). \tag{8}$$

The *tensor* form factors can also be defined in the following way:

$$\begin{aligned}\langle T(P,\lambda)|\bar{q}\,\sigma^{\mu\nu}\gamma_5 b|\bar{B}_q(P_B)\rangle &= -iA^{BT}(q^2)\left[e^{*\mu}_{(\lambda)}(P+P_B)^\nu-(P+P_B)^\mu e^{*\nu}_{(\lambda)}\right]\\ &+iB^{BT}(q^2)\left[e^{*\mu}_{(\lambda)}q^\nu-q^\mu e^{*\nu}_{(\lambda)}\right]+2iC^{BT}(q^2)\frac{e^*_{(\lambda)}q}{m_{B_q}^2-m_T^2}\left[P^\mu q^\nu-q^\mu P^\nu\right].\end{aligned} \tag{9}$$

We then find that

$$\begin{aligned}T_1^{BT}(q^2)&=A^{BT}(q^2),\\ T_2^{BT}(q^2)&=A^{BT}(q^2)-\frac{q^2}{m_{B_q}^2-m_T^2}B^{BT}(q^2),\\ T_3^{BT}(q^2)&=B^{BT}(q^2)+C^{BT}(q^2).\end{aligned} \tag{10}$$

3. To calculate the form factors, we consider the following two-point correlation functions, which are sandwiched between the vacuum and transversely polarized T meson with $\lambda=\pm1$:

$$\begin{aligned}&i\int d^4x e^{iqx}\langle T(P,\lambda=1)|T[\bar{q}_1(x)\gamma_\mu(1-\gamma_5)b(x)\,j^\dagger_{B_{q_2}}(0)]|0\rangle\\ &=-\mathbf{A_1}(p_B^2,q^2)e^{*(\perp)}_\mu+\mathbf{A_2}(p_B^2,q^2)(e^{*(\perp)}q)(2P+q)_\mu+\mathbf{A}(p_B^2,q^2)\frac{e^{*(\perp)}q}{q^2}q_\mu\\ &\quad -i\mathbf{V}(p_B^2,q^2)\varepsilon_{\mu\nu\rho\sigma}e^{*\nu}_{(\perp)}q^\rho P^\sigma,\end{aligned} \tag{11}$$

and

$$\begin{aligned}&i\int d^4x e^{iqx}\langle T(P,\lambda=1)|T[\bar{q}_1(x)\sigma_{\mu\nu}\gamma_5 b(x)\,j^\dagger_{B_{q_2}}(0)]|0\rangle\\ &=-i\mathcal{A}(p_B^2,q^2)\{e^{*(\perp)}_\mu(2P+q)_\nu-e^{*(\perp)}_\nu(2P+q)_\mu\}\\ &\quad +i\mathcal{B}(p_B^2,q^2)\{e^{*(\perp)}_\mu q_\nu-e^{*(\perp)}_\nu q_\mu\}+2i\mathcal{C}(p_B^2,q^2)\frac{e^{*(\perp)}q}{m_{B_q}^2-m_T^2}\{P_\mu q_\nu-q_\mu P_\nu\},\end{aligned} \tag{12}$$

where

$$e^{*(\perp)}_\mu\equiv\frac{1}{\sqrt{2}}\varepsilon^*_\mu(\pm1)\frac{\varepsilon^*(0)\cdot q}{m_B},$$

$p_B^2 = (P+q)^2$, P is the momentum of the T meson, and $j_{B_{q_2}} = i\bar{q}_2\gamma_5 b$ (with $q_{2(1)} \equiv u, d$ or s) is the interpolating current for the B_{q_2} meson, so that

$$\langle 0|j_{B_{q_2}}(0)|\bar{B}_{q_2}(p_B)\rangle = \frac{f_{B_{q_2}} m_{B_{q_2}}^2}{m_b + m_{q_2}}\,. \tag{13}$$

In the region of sufficiently large virtualities, i.e., $m_b^2 - p_B^2 \gg \Lambda_{\rm QCD} m_b$ where q^2 is small and positive, the operator product expansion is applicable in (11) and (12), so that for an energetic T meson the correlation functions defined in (11) and (12) can be generally represented in terms of the LCDAs of the T meson (with $\Gamma \equiv$ the vector, axial-vector, or tensor current):

$$\begin{aligned} & i\int d^4x e^{iqx}\langle T(P,\lambda=1)|T[\bar{q}_1(x)\Gamma b(x)\, j_B^\dagger(0)]|0\rangle \\ &\quad = \int_0^1 du \frac{-i}{(q+k)^2 - m_b^2}\mathrm{Tr}\Big[\Gamma(\not{q} + \not{k} + m_b)\gamma_5 M_\perp^T(\lambda=1)\Big]\Bigg|_{k=uEn_-} + \mathcal{O}\bigg(\frac{m_T^2}{E^2}\bigg)\,, \end{aligned} \tag{14}$$

where $E = |\vec{P}|$, $P^\mu = En_-^\mu + m_T^2 n_+^\mu/(4E) \simeq En_-^\mu$ with two light-like vectors $n_-^\mu = (1,0,0,-1)$ and $n_+^\mu = (1,0,0,1)$ satisfying $n_- n_+ = 2$ and $n_-^2 = n_+^2 = 0$. Here $M_\perp^T$ is the transverse projector which is discussed as follows. We assign the momentum of the q_1-quark in the tensor meson as

$$k^\mu = uEn_-^\mu + k_\perp^\mu + \frac{k_\perp^2}{4uE} n_+^\mu\,, \tag{15}$$

where $k_\perp$ is of order $\Lambda_{\rm QCD}$, E is of order m_b, and u is the momentum fraction carried by the q_1-quark in the meson. In (14), to calculate contributions in the momentum space, we use the following substitution (with the $k_\perp^2$ term omitted):

$$x^\mu \to -i\frac{\partial}{\partial k_\mu} \simeq -i\bigg(\frac{n_+^\mu}{2E}\frac{\partial}{\partial u} + \frac{\partial}{\partial k_{\perp\,\mu}}\bigg)\,, \tag{16}$$

to the Fourier transform for

$$\begin{aligned} \langle T(P,\lambda)|\bar{q}_\alpha^1(x)\, q_\delta^2(0)|0\rangle &= -\frac{i}{4}\int_0^1 du\, e^{iuPx}\bigg\{ f_T m_T^2\bigg[\not{P}\,\frac{\epsilon_{\mu\nu}^{*(\lambda)} x^\mu x^\nu}{(Px)^2}\,\Phi_\parallel^T(u) - \not{x}\frac{\epsilon_{\mu\nu}^{*(\lambda)} x^\mu x^\nu}{2(Px)^3} m_T^2 \bar{g}_3(u) \\ &+ \bigg(\frac{\epsilon_{\mu\nu}^{*(\lambda)} x^\nu}{Px} - P_\mu \frac{\epsilon_{\nu\beta}^{*(\lambda)} x^\nu z^\beta}{(Px)^2}\bigg)\gamma^\mu\, g_v(u) + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu \epsilon_{(\lambda)}^{*\nu\beta} x_\beta P^\rho x^\sigma \gamma_5 \frac{1}{Px}\, g_a(u)\bigg] \\ &- \frac{i}{2} f_T^\perp m_T\bigg[\frac{\sigma^{\mu\nu}\left(\epsilon_{\mu\beta}^{(\lambda)*} x^\beta P_\nu - \epsilon_{\nu\beta}^{(\lambda)*} x^\beta P_\mu\right)}{Px}\Phi_\perp^T(u) + \frac{\sigma^{\mu\nu}(P_\mu x_\nu - P_\nu x_\mu) m_T^2 \epsilon_{\rho\beta}^{(\lambda)*} x^\rho x^\beta}{(Px)^3}\,\bar{h}_t(u) \\ &+ \sigma^{\mu\nu}\left(\epsilon_{\mu\beta}^{(\lambda)*} x^\beta x_\nu - \epsilon_{\nu\beta}^{(\lambda)*} x^\beta x_\mu\right)\frac{m_T^2}{2(Px)^2}\,\bar{h}_3(u) + \epsilon_{\mu\nu}^{*(\lambda)} x^\mu x^\nu \frac{m_T^2}{Px}\, h_s(u)\bigg] + \mathcal{O}(x^2)\bigg\}_{\delta\alpha}\,, \end{aligned} \tag{17}$$

where $x^2 \neq 0$ and we have

$$\begin{aligned} \bar{g}_3(u) &= g_3(u) + \Phi_\parallel^T - 2g_v(u), \\ \bar{h}_t(u) &= h_t(u) - \frac{1}{2}\Phi_\perp^T(u) - \frac{1}{2}h_3(u), \\ \bar{h}_3(u) &= h_3(u) - \Phi_\perp^T(u). \end{aligned} \tag{18}$$

$\Phi_\parallel^T, \Phi_\perp^T$ are of twist-2, g_v, g_a, h_t, h_s are of twist-3, and g_3, h_3 are of twist-4. To leading-twist accuracy, we will use the following approximations for the relevant two-parton LCDAs in the present study [1]:

$$\Phi^T_{\parallel,\perp}(u,\mu) \simeq 6u(1-u)a_1^{(\parallel,\perp),T}(\mu)C_1^{3/2}(2u-1), \tag{19}$$

$$g_v(u) \simeq \int_0^u dv\, \frac{\Phi_\parallel^T(v)}{\bar v} + \int_u^1 dv\, \frac{\Phi_\parallel^T(v)}{v}\,,$$

$$g_a(u) \simeq 2\bar u \int_0^u dv\, \frac{\Phi_\parallel^T(v)}{\bar v} + 2u\int_u^1 dv\, \frac{\Phi_\parallel^T(v)}{v}\,. \tag{20}$$

In general, the G-parity violating parameters $a_0^{(\parallel,\perp),T}$ vanish because the tensor meson cannot be produced from the local $V-A$ current. Consequently, from (17) we can obtain the light-cone projection for the T meson in momentum space,

$$M^T_{\delta\alpha} = M_{\delta\alpha}{}^T_\parallel(\lambda=0) + M_{\delta\alpha}{}^T_\perp(\lambda=1) + M_{\delta\alpha}{}^T_\perp(\lambda=2)\,, \tag{21}$$

where $M^T_{\delta\alpha\parallel}$ and $M^T_{\delta\alpha\perp}$ are the longitudinal and transverse projectors, respectively. The transverse projector $M_{\delta\alpha}{}^T_\perp(\lambda=1)$, which is relevant here, is given by

$$\begin{aligned} M_\perp^T(\lambda=\pm1) = -i\frac{f_T^\perp}{4}E\left[\epsilon^{*(\lambda)}_{\perp\mu\alpha}n_+^\alpha\left(\frac{m_T}{2E}\right)\right]\Bigg\{&\gamma^\mu\, \not{n}_-\,\Phi_\perp^T(u) \\ &+\frac{f_T}{f_T^\perp}\frac{m_T}{E}\left[\gamma^\mu g_v(u) - E\int_0^u dv\left(2\Phi_\parallel^T(v)-g_v(v)\right)\not{n}_-\frac{\partial}{\partial k_{\perp\mu}}\right. \\ &\left. -\ i\varepsilon_{\mu\nu\rho\sigma}\,\gamma^\nu n_-^\rho\gamma_5\left(n_+^\sigma\frac{g_a'(u)}{4} - E\frac{g_a(u)}{2}\frac{\partial}{\partial k_{\perp\sigma}}\right)\right] + \mathcal{O}\left(\frac{m_T^2}{E^2}\right)\Bigg\}, \end{aligned} \tag{22}$$

where

$$\epsilon^{*(\lambda)}_{\perp\mu\nu}n_+^\nu\left(\frac{m_T}{2E}\right) = \sqrt{\frac{1}{2}}\,\varepsilon^*_\mu(\pm1)\delta_{\lambda,\pm1}\,. \tag{23}$$

From the expansion of $M_{\delta\alpha}{}^T_\perp(\lambda=1)$, one can find that the contributions arising from $g_v, 2\Phi_\parallel^T - g_v$, and g_a are suppressed by m_T/E relative to the term with $\Phi_\perp$, i.e., the expansion parameter in the light-cone sum rules should be m_T/m_b, rather than the twist. Note that in (14) the derivative with respect to the transverse momentum acts on the hard scattering amplitude before the collinear approximation is taken.

At the quark-gluon level, after performing the integration of (14), the results up to $\mathcal{O}(m_T/m_b)$ read

$$\mathbf{A_1}^{\rm QCD} = \frac{m_B}{\varepsilon^*(0)\cdot q}\frac{m_b^2 f_T^\perp}{2}\int_0^1\frac{du}{u}\left\{\frac{1}{m_b^2-up_B^2-\bar u q^2}\left[\frac{m_b^2-q^2}{m_b^2}\Phi_\perp^T(u) + \left(\frac{m_T f_T}{m_b f_T^\perp}\right)2ug_v(u)\right]\right\}, \tag{24}$$

$$\mathbf{A_2}^{\rm QCD} = \frac{m_B}{\varepsilon^*(0)\cdot q}\frac{m_b^2 f_T^\perp}{2}\int_0^1 du\left\{\frac{1}{m_b^2-up_B^2-\bar u q^2}\frac{\Phi_\perp^T(u)}{m_b^2} + \frac{m_T f_T}{m_b f_T^\perp}\frac{2\Phi_{tt}(u)}{(m_b^2-up_B^2-\bar u q^2)^2}\right\}, \tag{25}$$

$$\mathbf{A}^{\rm QCD} = -q^2\mathbf{A_2}^{\rm QCD}, \tag{26}$$

$$\mathbf{V}^{\rm QCD} = \frac{m_B}{\varepsilon^*(0)\cdot q} m_b^2 f_T^\perp \int_0^1 du \Bigg\{ \frac{1}{m_b^2 - u p_B^2 - \bar u q^2} \frac{\Phi_\perp^T(u)}{m_b^2} + \frac{m_T f_T}{m_b f_T^\perp} \frac{g_a(u)}{(m_b^2 - u p_B^2 - \bar u q^2)^2} \Bigg\}, \tag{27}$$

$$\mathcal{A}^{\rm QCD} = \frac{m_B}{\varepsilon^*(0)\cdot q} \frac{m_b f_T^\perp}{2} \int_0^1 du \Bigg\{ \frac{1}{m_b^2 - u p_B^2 - \bar u q^2} \Bigg[\Phi_\perp^T(u) + \frac{m_T f_T}{m_b f_T^\perp} \Bigg(u g_v(u) + \Phi_{tt}(u) + \frac{g_a(u)}{2} \Bigg) \Bigg] + \frac{m_T f_T}{2 m_b f_T^\perp} \frac{(m_b^2 + q^2)}{(m_b^2 - u p_B^2 - \bar u q^2)^2} g_a(u) \Bigg\}, \tag{28}$$

$$\mathcal{B}^{\rm QCD} = \frac{m_B}{\varepsilon^*(0)\cdot q} \frac{m_b f_T^\perp}{2} \int_0^1 du \Bigg\{ \frac{1}{m_b^2 - u p_B^2 - \bar u q^2} \Bigg[\Phi_\perp^T(u) + \frac{m_T f_T}{m_b f_T^\perp} \Bigg((u-2) g_v(u) + \Phi_{tt}(u) + \Big(\frac{1}{2} - \frac{1}{u}\Big) g_a(u) \Bigg) \Bigg] + \frac{m_T f_T}{m_b f_T^\perp} \Bigg[m_b^2 - (m_b^2 - q^2) \Bigg(\frac{1}{2} - \frac{1}{u} \Bigg) \Bigg] \frac{g_a(u)}{(m_b^2 - u p_B^2 - \bar u q^2)^2} \Bigg\}, \tag{29}$$

$$\mathcal{C}^{\rm QCD} = \frac{m_B}{\varepsilon^*(0)\cdot q} \frac{f_T m_T m_{B_{q_2}}^2}{2} \int_0^1 du \Bigg\{ \frac{1}{(m_b^2 - u p_B^2 - \bar u q^2)^2} \Bigg[2\Phi_{tt}(u) - g_a(u) \Bigg] \Bigg\}, \tag{30}$$

where $\Phi_{tt}(u) \equiv \int_0^u dv\, (2\Phi_\parallel(v) - g_v(v))$ and $\bar u \equiv 1 - u$.

As an example, the form factor A_1 for the $B \to T$ transition can be further approximated with the help of quark-hadron duality:

$$A_1(q^2) \cdot \frac{m_{B_{q_2}} + m_T}{m_{B_{q_2}}^2 - p_B^2} \cdot \frac{m_{B_{q_2}}^2 f_{B_{q_2}}}{m_b + m_{q_2}} = \frac{1}{\pi} \int_{m_b^2}^{s_0} \frac{{\rm Im}\mathbf{A_1}^{\rm QCD}(s,q^2)}{s - p_B^2} ds\,, \tag{31}$$

where s_0 is the excited state threshold. After applying the Borel transform $p_B^2 \to M^2$ [11] to the above equation, we obtain

$$A_1(q^2) = \frac{(m_b + m_{q_2})}{(m_{B_{q_2}} + m_T) m_{B_{q_2}}^2 f_{B_{q_2}}} e^{m_{B_{q_2}}^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} e^{s/M^2} {\rm Im}\mathbf{A_1}^{\rm QCD}(s,q^2) ds\,. \tag{32}$$

We summarize the light-cone sum rule results as follows,

$$A_1^{B_{q_2}T}(q^2) = d \frac{(m_b + m_{q_2}) m_b^2 f_T^\perp}{(m_{B_{q_2}} + m_T) m_{B_{q_2}}^2 f_{B_{q_2}}} e^{(m_{B_{q_2}}^2 - m_b^2)/M^2} \times \int_0^1 \frac{du}{u} e^{\bar u (q^2 - m_b^2)/(u M^2)} \theta[c(u,s_0)] \Bigg[\Phi_\perp^T(u) \frac{m_b^2 - q^2}{2 u m_b^2} + \frac{m_{B_{q_2}} + m_T}{m_{B_{q_2}}} \frac{m_T f_T}{m_b f_T^\perp} g_v(u) \Bigg], \tag{33}$$

$$A_2^{B_{q_2}T}(q^2) = d \frac{(m_b + m_{q_2})(m_{B_{q_2}} + m_T) f_T^\perp}{2 m_{B_{q_2}}^2 f_{B_{q_2}}} e^{(m_{B_{q_2}}^2 - m_b^2)/M^2} \int_0^1 \frac{du}{u} e^{\bar u (q^2 - m_b^2)/(u M^2)} \Bigg[\Phi_\perp^T(u) \theta[c(u,s_0)] + \frac{m_{B_{q_2}}}{m_{B_{q_2}} + m_T} \frac{2 m_T m_b f_T}{u M^2 f_T^\perp} \Phi_{tt}(u) \Bigg(\theta[c(u,s_0)] + u M^2 \delta[c(u,s_0)] \Bigg) \Bigg], \tag{34}$$

$$A_0^{B_{q_2}T}(q^2) = A_3^{B_{q_2}T}(q^2) + d \frac{q^2 (m_b + m_{q_2}) f_T^\perp}{4 m_{B_{q_2}}^2 m_T f_{B_{q_2}}} e^{(m_{B_{q_2}}^2 - m_b^2)/M^2} \int_0^1 du \Bigg\{ \frac{1}{u} e^{\bar u (q^2 - m_b^2)/(u M^2)} \times \Bigg[\Phi_\perp^T(u) \theta[c(u,s_0)] + \frac{2 m_T m_b f_T}{u M^2 f_T^\perp} \Phi_{tt}(u) \Bigg(\theta[c(u,s_0)] + u M^2 \delta[c(u,s_0)] \Bigg) \Bigg] \Bigg\}, \tag{35}$$

$$V^{B_{q_2}T}(q^2) = d\frac{(m_b+m_{q_2})(m_{B_{q_2}}+m_T)f_T^\perp}{2m_{B_{q_2}}^2 f_{B_{q_2}}} e^{(m_{B_{q_2}}^2-m_b^2)/M^2}\int_0^1 \frac{du}{u} e^{\bar u(q^2-m_b^2)/(uM^2)}\Bigg[\Phi_\perp^T(u)\theta[c(u,s_0)]$$
$$+\frac{m_{B_{q_2}}}{m_{B_{q_2}}+m_T}\frac{m_T m_b f_T}{uM^2 f_T^\perp} g_a(u)\Bigg(\theta[c(u,s_0)]+uM^2\delta[c(u,s_0)]\Bigg)\Bigg], \tag{36}$$

$$T_1^{B_{q_2}T}(q^2) = d\frac{(m_b+m_{q_2})m_b f_T^\perp}{2m_{B_{q_2}}^2 f_{B_{q_2}}} e^{(m_{B_{q_2}}^2-m_b^2)/M^2}$$
$$\times \int_0^1 \frac{du}{u} e^{\bar u(q^2-m_b^2)/(uM^2)}\Bigg\{\theta[c(u,s_0)]\Bigg[\Phi_\perp^T(u)+\frac{m_T f_T}{m_b f_T^\perp}\Big(u g_v(u)+\Phi_{tt}(u)+\frac{g_a(u)}{2}\Big)\Bigg]$$
$$+\frac{m_T f_T}{2m_b f_T^\perp}(m_b^2+q^2)g_a(u)\Bigg(\frac{\theta[c(u,s_0)]}{uM^2}+\delta[c(u,s_0)]\Bigg)\Bigg\}, \tag{37}$$

$$T_2^{B_{q_2}T}(q^2) = T_1^{B_{q_2}T}(q^2) - \frac{q^2}{m_{B_{q_2}}^2-m_T^2}B^{BT}(q^2), \qquad T_3^{B_{q_2}T}(q^2) = B^{BT}(q^2)+C^{BT}(q^2)\,, \tag{38}$$

with

$$B^{BT}(q^2) = d\frac{(m_b+m_{q_2})m_b f_T^\perp}{2m_{B_{q_2}}^2 f_{B_{q_2}}} e^{(m_{B_{q_2}}^2-m_b^2)/M^2}\int_0^1 \frac{du}{u} e^{\bar u(q^2-m_b^2)/(uM^2)}\Bigg\{\theta[c(u,s_0)]\Bigg[\Phi_\perp^T(u)+\frac{m_T f_T}{m_b f_T^\perp}$$
$$\times\Bigg((u-2)g_v(u)+\Phi_{tt}(u)+\Big(\frac{1}{2}-\frac{1}{u}\Big)g_a(u)\Bigg)\Bigg]$$
$$+\frac{m_T f_T}{m_b f_T^\perp}\Bigg[m_b^2-(m_b^2-q^2)\Bigg(\frac{1}{2}-\frac{1}{u}\Bigg)\Bigg]g_a(u)\Bigg(\frac{\theta[c(u,s_0)]}{uM^2}+\delta[c(u,s_0)]\Bigg)\Bigg\}, \tag{39}$$

$$C^{BT}(q^2) = d\frac{(m_b+m_{q_2})m_T f_T}{2f_{B_{q_2}}} e^{(m_{B_{q_2}}^2-m_b^2)/M^2}$$
$$\times\int_0^1 \frac{du}{u} e^{\bar u(q^2-m_b^2)/(uM^2)}\Bigg(2\Phi_{tt}(u)-g_a(u)\Bigg)\Bigg(\frac{\theta[c(u,s_0)]}{uM^2}+\delta[c(u,s_0)]\Bigg)\,, \tag{40}$$

where $A_3^{B_{q_2}T}(q^2)$ is defined by (8), $c(u,s_0)=us_0-m_b^2+(1-u)q^2$, and

$$d=\frac{2m_T}{m_B}\frac{1}{1-q^2/m_B^2}.$$

4. The wave functions of the isoscalar tensor states $f_2(1270)$ and $f_2'(1525)$ are

$$f_2(1270)=\frac{1}{\sqrt{2}}(f_2^u+f_2^d)\cos\theta_{f_2}+f_2^s\sin\theta_{f_2}\ , \quad f_2'(1525)=\frac{1}{\sqrt{2}}(f_2^u+f_2^d)\sin\theta_{f_2}-f_2^s\cos\theta_{f_2}\ ,$$

with $f_2^u\equiv u\bar u$ and likewise for $f_2^{d,s}$. Because $\pi\pi$ is the dominant decay mode of $f_2(1270)$, and $f_2'(1525)$ decays are predominated by the $K\bar K$ mode (see Ref. [12]), the mixing angle should be small. It was found that $\theta_{f_2}=7.8^\circ$ [13] and $(9\pm1)^\circ$ [12]. Therefore, we assume $f_2(1270)$ is primarily a $(u\bar u+d\bar d)/\sqrt{2}$ state, while $f_2'(1525)$ is predominated by $s\bar s$. We summarize the relevant parameters in Table I, where the $f_{f_2(1270)}$ and $f_{K_2^*(1430)}$ were also studied in [14] and [15], respectively. We neglect

TABLE I. Input parameters. $f_T^{(\perp)}(\mu)$ and $a_1^{(\parallel,\perp),T}(\mu)$ are given at the scale $\mu = 1\,\text{GeV}$.

Light tensor mesons [1]			
T	f_T [MeV]	$f_T^\perp$ [MeV]	$a_1^{\parallel,T}, a_1^{\perp,T}$
$f_2(1270)$	102 ± 6	117 ± 25	$\frac{5}{3}$
$f_2'(1525)$	126 ± 4	65 ± 12	$\frac{5}{3}$
$a_2(1320)$	107 ± 6	105 ± 21	$\frac{5}{3}$
$K_2^*(1430)$	118 ± 5	77 ± 14	$\frac{5}{3}$
Strange quark mass (GeV), pole b-quark mass (GeV), and couplings [16]			
$m_s(2\,\text{GeV})$	$m_{b,pole}$	$\alpha_s(1\text{ GeV})$	$\alpha_s(2.2\text{ GeV})$
0.09 ± 0.01	4.85 ± 0.05	0.495	0.287
Effective $B_{(s)}$ decay constants [16, 17]			
$\bar{f}_B$ [MeV]		$\bar{f}_{B_s}$ [MeV]	
145 ± 10		165 ± 10	

the possible corrections due to m_u and m_d. The pole b quark mass is used and the scale-dependent parameters are evaluated at the factorization scale $\mu_f = \sqrt{m_{B_q}^2 - m_{b,pole}^2}$. In the numerical analysis for the form-factor sum rules, we choose the excited state threshold of the B meson to be $s_0 = 35.5 \pm 2.0\text{ GeV}^2$ and the Borel window $6.0\text{ GeV}^2 < M^2 < 12.0\text{ GeV}^2$, which is consistent with my previous study [16]. We use the *effective* B decay constants $\bar{f}_B = 145 \pm 10$ MeV and $\bar{f}_{B_s} \simeq 165 \pm 11$ MeV [16], which are obtained from the QCD sum rules of the B decay constants without including the radiative corrections to the coefficient of the unity operator. The former is in accordance with [17]. In [16], we have checked that, using the present values of f_B and m_b in the LCSR of $B \to \rho$ form factors of the same order, we can get results which are in good agreement with those given in [18] where the radiative corrections are included. It was found that the radiative corrections to form factors can be canceled if one adopts the f_B sum rule result with the same order of α_s-corrections in the calculation [18, 19]. Therefore, the radiative corrections might be negligible, so we do not include them.

We also assume that this set of parameters can apply to form factors with various q^2. Our results from fitting the q^2-dependence of form factors within the range $0 \le q^2 \le 6\text{ GeV}^2$ are exhibited in Tables II and III, where the q^2-dependence is parameterized in the three-parameter form:

$$F^{B_qT}(q^2) = \frac{F^{B_qT}(0)}{1 - a(q^2/m_{B_q}^2) + b(q^2/m_{B_q}^2)^2}\,, \tag{41}$$

with $F^{B_qT} \equiv A_{0,1,2}^{B_qT}$, V^{B_qT} or $T_{1,2,3}^{B_qT}$. For simplicity, we do not show the theoretical errors for the parameters a and b. The magnitude of $T_3^{B_qT}(0)$ is small in general due to the fact that $B^{B_qT}(0)$ and $C^{B_qT}(0)$, defined by (39) and (40), have similar magnitudes but opposite signs.

5. In summary, using the recent analysis of tensor meson distribution amplitudes [1], we have

TABLE II. $B \to$ tensor meson form factors obtained in the LCSR calculation are fitted to the 3-parameter form in (41). The error for $F^{BT}(0)$ is due to the variations of the Borel mass, decay constants, strange quark mass, and pole b quark mass, which are then added in quadrature.

F	$[F^{Ba_2}(0), a, b]$	$[F^{BK_2^*}(0), a, b]$	$[F^{Bf_2}(0), a, b]$
A_1	$[0.14 \pm 0.02, 1.21, 0.52]$	$[0.14 \pm 0.02, 1.23, 0.49]$	$[0.14 \pm 0.02, 1.20, 0.54]$
A_2	$[0.09 \pm 0.02, 1.27, 2.04]$	$[0.05 \pm 0.02, 1.32, 14.9]$	$[0.10 \pm 0.02, 1.45, 1.58]$
A_0	$[0.21 \pm 0.04, 1.19, -0.26]$	$[0.25 \pm 0.04, 1.57, 0.10]$	$[0.20 \pm 0.04, 0.99, -0.34]$
V	$[0.18 \pm 0.02, 2.10, 1.50]$	$[0.16 \pm 0.02, 2.08, 1.50]$	$[0.18 \pm 0.02, 2.10, 1.51]$
T_1	$[0.15 \pm 0.02, 2.09, 1.50]$	$[0.14 \pm 0.02, 2.07, 1.50]$	$[0.15 \pm 0.02, 2.10, 1.50]$
T_2	$[0.15 \pm 0.02, 1.21, 0.39]$	$[0.14 \pm 0.02, 1.22, 0.35]$	$[0.14 \pm 0.02, 1.20, 0.41]$
T_3	$[0.04 \pm 0.02, 2.14, 23.6]$	$[0.01^{+0.02}_{-0.01}, \quad 9.91, 276]$	$[0.06 \pm 0.02, 1.04, 6.36]$

TABLE III. Same as Table II except for $B_s \to$ tensor meson transitions.

F	$[F^{B_sK_2^*}(0), a, b]$	$[F^{B_sf_2'}(0), a, b]$
A_1	$[0.12 \pm 0.02, 1.23, 0.48]$	$[0.13 \pm 0.02, 1.25, 0.47]$
A_2	$[0.05 \pm 0.02, 1.32, 14.9]$	$[0.03 \pm 0.02, 4.71, 105]$
A_0	$[0.22 \pm 0.04, 1.57, 0.10]$	$[0.25 \pm 0.04, 1.72, 0.31]$
V	$[0.15 \pm 0.02, 2.08, 1.50]$	$[0.15 \pm 0.02, 2.06, 1.49]$
T_1	$[0.13 \pm 0.02, 2.07, 1.49]$	$[0.13 \pm 0.02, 2.06, 1.49]$
T_2	$[0.13 \pm 0.02, 1.22, 0.35]$	$[0.13 \pm 0.02, 1.23, 0.32]$
T_3	$[0.01^{+0.02}_{-0.01}, \quad 9.91, 276]$	$[0.00^{+0.02}_{-0.00}, \quad —, \quad —]$

calculated the form factors of B decays into tensor mesons with the light-cone sum rule approach. We have fitted the q^2-dependence of the form factors in the range $0 \leq q^2 \leq 6$ GeV^2. Owing to the G-parity, the two-parton light-cone distribution amplitudes of the tensor mesons are antisymmetric under the interchange of its quark and anti-quark momentum fractions in the SU(3) limit. The sum rule results for form factors are sensitive to the light-cone distribution amplitudes. The expansion parameter in the light-cone sum rules is m_T/m_b, rather than the twist. For the resulting sum rules, we have included the terms up to the order of m_T/m_b in the light-cone expansion. The results could be further improved with more precise parameters describing the distribution amplitudes and by including $\mathcal{O}(\alpha_s)$ corrections.

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[1] H. Y. Cheng, Y. Koike and K. C. Yang, Phys. Rev. D **82**, 054019 (2010).
[2] H. Y. Cheng and K. C. Yang, arXiv:1010.3309 [hep-ph].
[3] N. Isgur, D. Scora, B. Grinstein, and M.B. Wise, Phys. Rev. D **39**, 799 (1989).
[4] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D **69**, 074025 (2004).
[5] H. Hatanaka and K. C. Yang, Phys. Rev. D **79**, 114008 (2009).
[6] H. Hatanaka and K. C. Yang, Eur. Phys. J. C **67**, 149 (2010).
[7] W. Wang, arXiv:1008.5326 [hep-ph].
[8] A. S. Safir, Eur. Phys. J. direct C **3**, 15 (2001).
[9] V. M. Braun and N. Kivel, Phys. Lett. B **501**, 48 (2001).
[10] E. R. Berger, A. Donnachie, H. G. Dosch and O. Nachtmann, Eur. Phys. J. C **14**, 673 (2000).
[11] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448, 519.
[12] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).
[13] D.M. Li, H. Yu, and Q.X. Shen, J. Phys. G **27**, 807 (2001)
[14] T. M. Aliev and M. A. Shifman, Phys. Lett. B **112**, 401 (1982).
[15] T. M. Aliev, K. Azizi and V. Bashiry, J. Phys. G **37**, 025001 (2010).
[16] K. C. Yang, Phys. Rev. D **78**, 034018 (2008).
[17] A. Khodjamirian and R. Ruckl, Adv. Ser. Direct. High Energy Phys. **15**, 345 (1998).
[18] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014029 (2005).
[19] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).